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Generalized electromagnetic fields in a chiral medium

P S Bisht[1], **Jivan Singh**[2] **and O P S Negi**[1,3]

[1] Department of Physics, Kumaun University, Soban Singh Jeena Campus, Almora-263601 (Uttarakhand), India
[2] Department of Physics, Govt. Post Graduate College, Pithoragarh (Uttarakhand), India

E-mail: ps_bisht123@rediffmail.com, jgaria@indiatimes.com and ops_negi@yahoo.co.in

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Abstract
The time-dependent Dirac–Maxwell's equations in the presence of electric and magnetic sources are reformulated in a chiral medium, and the solutions for the classical problem are obtained in a unique, simple and consistent manner. The quaternion reformulation of generalized electromagnetic fields in the chiral medium has also been discussed in a compact, simple and consistent manner.

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1. Introduction

Physicists were fascinated about magnetic monopoles since its ingenious idea was put forward by Dirac [1] and also by Saha [2]. So many attempts [3, 4] were made for the experimental verification of the conclusive existence of magnetic monopoles and after the failure of attempts, the literature [5–7] turned partially negative casting doubts on the existence of such particles. The work of the Schwinger [8] was the first exception to the argument against the existence of monopoles. At the same time, so many paradoxes were related to the theory of pure Abelian monopoles, as Dirac's veto, wrong spin-statistics connection [9] and many others [10, 11]. Several problems were soon resolved by the invention of dyons [12–15], particles carrying the simultaneous existence of electric and magnetic charges. Fresh interest in this subject was enhanced by the idea given by t'Hooft [16] and Polyakov [17] showing that monopoles are the intrinsic parts of grand unified theories. The Dirac monopole is an elementary particle but the t'Hooft–Polyakov monopole [16, 17] is a complicated extended object having a definite mass and finite size inside of which massive fields play a role in providing a smooth structure and outside it they vanish rapidly leaving the field configuration identical to the Abelian Dirac monopole. Julia and Zee [18] have extended the idea of t' Hooft [16] and Polyakov [17] to construct the classical solutions for non-Abelian dyon. On the other hand, Prasad and

[3] Address from 01 July 2007 to 17 September 2007: Institute of Theoretical Physics, Chinese Academy of Sciences, Hai Dian Qu Zhong Guan Chun Dong Lu, 55 Hao, Beijing-100080, People's Republic of China.

Sommerfield [19, 20] have derived the analytic stable solutions for the non-Abelian monopoles and dyons of finite mass by keeping the symmetry of vacuum broken but letting the self-interaction of Higgs field approaching zero. Such solutions, satisfying Bogomolnys condition [21], are described as Bogomolny–Prasad–Sommerfield (BPS) monopoles. Kravchenko and coauthors [22, 23] discussed the Maxwell's equations in homogenous media and developed [24] the quaternionic reformulation of the time-dependent Maxwell's equations along with the classical solution of a moving source i.e. electron. They have also demonstrated [25] the electromagnetic fields in the chiral medium along with their quaternionic reformulation in a simple and consistent manner. Recently, we have extended the work of Kravchenko [22, 23] and reformulated [26] the Maxwell–Dirac's equation in a homogenous (isotropic) medium along with their quaternionic forms in a unique and consistent way [27]. We have also described [28] the time-harmonic Maxwell's equations for generalized fields of dyons in a consistent manner. Keeping these facts in mind, in this paper, we have obtained time-dependent generalized Dirac–Maxwell's (GDM) equations in the presence of electric and magnetic sources in a chiral and homogenous (isotropic) medium. Our results generalize for instance to the results those have obtained earlier by Grudsky *et al* [25] where the approach is strongly based on the use of quaternion analyticity, which permits a more compact and simple formalism. It is emphasized that the quantum equations and the equation of motion derived therein reproduce the dynamics of electric charge similar to the theory described by Kravchenko [22, 23] in the absence of magnetic monopole or vice versa.

2. Generalized Maxwell's equation of dyons in an isotropic medium

Considering the existence of magnetic monopoles, Dirac [1] generalized the Maxwell's equations to accommodate sources carrying both magnetic and electric charges. We [26] have reformulated these equations namely generalized Dirac–Maxwell's (GDM) equations in the homogenous (isotropic) medium in the following manner i.e.,

$$\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \frac{\rho_e}{\epsilon} \\
\vec{\nabla} \cdot \vec{B} &= \mu \rho_m \\
\vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} - \frac{\vec{j}_m}{\epsilon} \\
\vec{\nabla} \times \vec{B} &= \frac{1}{v^2}\frac{\partial \vec{E}}{\partial t} + \mu \vec{j}_e
\end{aligned} \tag{1}$$

where ρ_e and ρ_m are respectively the electric and magnetic charge densities while $\vec{j}_e$ and $\vec{j}_m$ are the corresponding current densities, $\vec{D}$ is the electric induction vector, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field and ϵ_r and μ_r are defined respectively as relative permittivity and permeability in electric and magnetic fields.

The electric and magnetic fields of dyons are expressed in the homogenous medium in terms of two potentials as

$$\vec{E} = -\vec{\nabla}\phi_e - \frac{\partial \vec{C}}{\partial t} - \vec{\nabla} \times \vec{D} \tag{2}$$

$$\vec{B} = -\vec{\nabla}\phi_m - \frac{1}{v^2}\frac{\partial \vec{D}}{\partial t} + \vec{\nabla} \times \vec{C} \tag{3}$$

where $\{C^\mu\} = \{\phi_e, v\vec{C}\}$ and $\{D^\mu\} = \{v\phi_m, \vec{D}\}$ are the two four-potentials associated with electric and magnetic charges. Let us define the complex vector field $\vec{\psi}$ in the following form:

$$\vec{\psi} = \vec{E} - \mathrm{i}v\vec{B}. \tag{4}$$

Equations (2)–(4) establish the following relation between a generalized vector field and the components of the generalized four-potential as

$$\vec{\psi} = -\frac{\partial \vec{V}}{\partial t} - \vec{\nabla}\phi - \mathrm{i}v(\vec{\nabla} \times \vec{V}) \tag{5}$$

where $\{V_\mu\}$ is the generalized four-potential of dyons in the homogenous medium and is defined as

$$V_\mu = \{\phi, \vec{V}\} \tag{6}$$

i.e.

$$\phi = \phi_e - \mathrm{i}v\phi_m \tag{7}$$

and

$$\vec{V} = \vec{C} - \mathrm{i}\frac{\vec{D}}{v}. \tag{8}$$

Maxwell's field equation (1) may then be written in terms of generalized field $\vec{\psi}$ as

$$\vec{\nabla} \cdot \vec{\psi} = \frac{\rho}{\epsilon} \tag{9}$$

$$\vec{\nabla} \times \vec{\psi} = -\mathrm{i}v\left(\mu\vec{J} + \frac{1}{v^2}\frac{\partial \vec{\psi}}{\partial t}\right) \tag{10}$$

where ρ and $\vec{J}$ are the generalized charge and current source densities of dyons in the homogenous medium given by

$$\rho = \rho_e - \mathrm{i}\frac{\rho_m}{v} \tag{11}$$

$$\vec{J} = \vec{j_e} - \mathrm{i}v\vec{j_m}. \tag{12}$$

Taking the curl of equation (10) and using equation (9), let us introduce a new parameter $\vec{S}$ (i.e. the field current) as

$$\vec{S} = \Box\vec{\psi} = -\mu\frac{\partial \vec{J}}{\partial t} - \frac{1}{\epsilon}\vec{\nabla}\rho - \mathrm{i}v\mu(\vec{\nabla} \times \vec{J}) \tag{13}$$

where $\Box$ is the D'Alembertian operator.

In terms of components of complex potential, the Maxwell–Dirac's equations are written as

$$\Box\phi = v\mu\rho \tag{14}$$

$$\Box\vec{V} = \mu\vec{J}. \tag{15}$$

We may thus write the tensorial form of generalized Maxwell–Dirac's equations of dyons in the homogenous medium as

$$F_{\mu\nu,\nu} = j_\mu^e \tag{16}$$

$$F^d_{\mu\nu,\nu} = j_\mu^m. \tag{17}$$

Accordingly, the generalized field tensor of dyon is defined as

$$G_{\mu\nu} = F_{\mu\nu} - \mathrm{i}v F^d_{\mu\nu}. \tag{18}$$

It yields the following covariant form of a generalized field equation of dyon in the homogenous (isotropic) medium i.e.

$$G_{\mu\nu,\nu} = J_\mu \tag{19}$$

$$G^d_{\mu\nu,\nu} = 0. \tag{20}$$

3. Generalized Maxwell's equation for the homogenous medium in a quaternionic form

A quaternion is defined as

$$q = q_0 e_0 + q_1 e_1 + q_2 e_2 + q_3 e_3 \tag{21}$$

where q_0, q_1, q_2, q_3 are real numbers and called the components of the quaternion q and the quaternion units e_0, e_1, e_2, e_3 satisfy the following multiplication rules:

$$e_0^2 = 1 \qquad e_j e_k = -\delta_{jk} + \epsilon_{jkl} e_l \tag{22}$$

where δ_{jk} and ϵ_{jkl} $(j, k, l = 1, 2, 3$ and $e_0 = 1)$ are respectively the Kronecker delta and three-index Levi-Civita symbol. For any quaternion, there exists a quaternion conjugate

$$\overline{q} = q - q_1 e_1 - q_2 e_2 - q_3 e_3 = q_0 - \overrightarrow{q}. \tag{23}$$

A quaternion conjugate is an automorphism of ring of quaternion i.e.

$$(\overline{pq}) = (\overline{q})(\overline{p}). \tag{24}$$

The norm of a quaternion is given as

$$N(q) = q \cdot \overline{q} = \overline{q} \cdot q = q_0^2 + q_1^2 + q_2^2 + q_3^2 = |q|^2. \tag{25}$$

The inverse of a quaternion is also a quaternion

$$q^{-1} = \frac{\overline{q}}{|q|^2}. \tag{26}$$

The quaternionic form of differential operator may be defined as [27]

$$\boxdot = \left(-\frac{\mathrm{i}}{v}\partial_t + D\right)$$

and its quaternion conjugate as

$$\overline{\boxdot} = \left(-\frac{\mathrm{i}}{v}\partial_t - D\right) \tag{27}$$

where $D = \partial_1 e_1 + \partial_2 e_2 + \partial_3 e_3$. Defining the complex vector wavefunction $\overrightarrow{\psi}$ of a generalized electromagnetic field as [26]

$$\overrightarrow{\psi} = \overrightarrow{E} - \mathrm{i}v\overrightarrow{B}. \tag{28}$$

We can now express the generalized four-potential, four-current, electric and magnetic field in the quaternion analysis as [27]

$$V = -\mathrm{i}\frac{\phi}{v} + V_1 e_1 + V_2 e_2 + V_3 e_3 \tag{29}$$

$$J = -\mathrm{i}\rho v + J_1 e_1 + J_2 e_2 + J_3 e_3 \tag{30}$$

$$E = E_1 e_1 + E_2 e_2 + E_3 e_3 \tag{31}$$

$$B = B_1 e_1 + B_2 e_2 + B_3 e_3. \tag{32}$$

Operating second equation of (27) to equation (28) and using equation (22), we get

$$\overline{\boxdot}\psi = J. \tag{33}$$

Similarly on operating first equation of (27) to equation (30) and using equation (22), we get

$$\boxdot J = S. \tag{34}$$

Equations (33) and (34) are respectively the quaternion field equations associated with the generalized potential and current of dyons in the homogeneous (isotropic) medium. These equations are invariant under quaternion transformations as well as with homogeneous Lorentz transformations. Instead of four sets of differential equations of field equations we may write only one set of the quaternion differential equation in a compact, simple and consistent manner. We may reinterpret our results as that the algebra over the field of real numbers corresponds to four sets of differential equations given by equation (1), the algebra over the field of complex numbers corresponds to two sets of differential equations given by equations (9) and (10) while the algebra over the field of quaternion variables corresponds only to one set of differential equations given by equation (33). As such equations (33) and (34) are the quaternion forms of differential equation (9), (10) and (13) and may be visualized as the quaternion reformulation of generalized potential and current of dyons in the isotropic homogeneous medium in a compact, simple and consistent manner.

4. Generalized electromagnetic fields in chiral media

Let us consider the generalized Maxwell's equations for dyons in a homogenous (isotropic) medium as

$$\begin{aligned}
\vec{\nabla} \cdot \widetilde{E}(x) &= \frac{\rho_e(x)}{\epsilon} \\
\vec{\nabla} \cdot \widetilde{H}(x) &= \rho_m(x) \\
\vec{\nabla} \times \widetilde{E}(x) &= -\frac{\widetilde{j}_m(x)}{\epsilon} + \mathrm{i}\omega\widetilde{B}(x) \\
\vec{\nabla} \times \widetilde{H}(x) &= \mu\widetilde{j}_e(x) - \mathrm{i}\omega\widetilde{D}(x).
\end{aligned} \tag{35}$$

In the above equation, we have taken that the electric field $\vec{E}$ and magnetic field $\vec{B}$ are time harmonic. Considering electric and magnetic fields as under [28]

$$\vec{E}(x,t) = \mathrm{Re}(\vec{E}(x)\,\mathrm{e}^{-\mathrm{i}\omega t}) \qquad \vec{B}(x,t) = \mathrm{Re}(\vec{B}(x)\,\mathrm{e}^{-\mathrm{i}\omega t}). \tag{36}$$

Here we consider the Born–Fedorov constitutive equations [29–32],

$$\widetilde{D}(x) = \epsilon(\widetilde{E}(x) + \beta\,\nabla \times \widetilde{E}(x)) \qquad \widetilde{B}(x) = \mu(\widetilde{H}(x) + \beta\,\nabla \times \widetilde{H}(x)) \tag{37}$$

where ϵ, μ and β are respectively permittivity, permeability and chiral parameter. If the medium is isotropic, the Cartesian field components are given by

$$\begin{aligned}
\widetilde{D_x} &= \varepsilon \widetilde{E_x} + \varepsilon\beta \left(\frac{\partial \widetilde{E_z}}{\partial y} - \frac{\partial \widetilde{E_y}}{\partial z} \right) \\
\widetilde{D_y} &= \varepsilon \widetilde{E_y} + \varepsilon\beta \left(\frac{\partial \widetilde{E_x}}{\partial z} - \frac{\partial \widetilde{E_z}}{\partial x} \right) \\
\widetilde{D_z} &= \varepsilon \widetilde{E_z} + \varepsilon\beta \left(\frac{\partial \widetilde{E_y}}{\partial x} - \frac{\partial \widetilde{E_x}}{\partial y} \right) \\
\widetilde{B_x} &= \mu \widetilde{B_x} + \mu\beta \left(\frac{\partial \widetilde{B_z}}{\partial y} - \frac{\partial \widetilde{B_y}}{\partial z} \right) \\
\widetilde{B_y} &= \mu \widetilde{B_y} + \mu\beta \left(\frac{\partial \widetilde{B_x}}{\partial z} - \frac{\partial \widetilde{B_z}}{\partial x} \right) \\
\widetilde{B_z} &= \mu \widetilde{B_z} + \mu\beta \left(\frac{\partial \widetilde{B_y}}{\partial x} - \frac{\partial \widetilde{B_x}}{\partial y} \right).
\end{aligned} \tag{38}$$

Applying the condition given by (37) into third and fourth equations of equation (35) we get

$$\begin{aligned}
\nabla \times \widetilde{E}(x) &= -\frac{\widetilde{j_m}(x)}{\varepsilon} + \mathrm{i}\omega\mu(\widetilde{H}(x) + \beta\nabla \times \widetilde{H}(x)) \\
\nabla \times \widetilde{H}(x) &= -\widetilde{j_e}(x) - \mathrm{i}\omega\varepsilon(\widetilde{E}(x) + \beta\nabla \times \widetilde{E}(x)).
\end{aligned} \tag{39}$$

If we introduce the following notations

$$\begin{aligned}
\widetilde{E}(x) &= -\sqrt{\mu}\,\overrightarrow{E}(x) \qquad & \widetilde{H}(x) &= \sqrt{\varepsilon}\,\overrightarrow{H}(x) \\
\widetilde{j_e}(x) &= \sqrt{\varepsilon}\,\overrightarrow{j_e}(x) \qquad & \widetilde{j_m}(x) &= \sqrt{\varepsilon}\,\overrightarrow{j_m}(x),
\end{aligned} \tag{40}$$

the first and second equations of equations (35) and (39) reduce to the following differential equations as

$$\begin{aligned}
&\nabla . \overrightarrow{E} = -\frac{\rho_e(x)}{\epsilon\sqrt{\mu}} \qquad \nabla . \overrightarrow{H} = \frac{\rho_m(x)}{\sqrt{\varepsilon}} \\
&\nabla \times \overrightarrow{E}(x) = -\frac{\overrightarrow{j_m}}{\sqrt{\mu\varepsilon}} - \mathrm{i}\alpha(\overrightarrow{H}(x) + \beta\nabla \times \overrightarrow{H}(x)) \\
&\nabla \times \overrightarrow{H}(x) = \overrightarrow{j_e}(x) + \mathrm{i}\alpha(\overrightarrow{E}(x) + \beta\nabla \times \overrightarrow{E}(x))
\end{aligned} \tag{41}$$

where $\alpha = \frac{\omega}{v}$ is denoted as the wavenumber.

5. Maxwell's equation for dyons in a quaternionic chiral medium

Let us consider the following purely vectorial biquaternionic functions as

$$\overrightarrow{m}(x) = \overrightarrow{E}(x) + \mathrm{i}\overrightarrow{H}(x) \tag{42}$$

$$\overrightarrow{n}(x) = \overrightarrow{E}(x) - \mathrm{i}\overrightarrow{H}(x). \tag{43}$$

Operating the quaternionic differential operator D on equation (42), we get

$$D\overrightarrow{m}(x) = \frac{\rho(x)}{\varepsilon\sqrt{\mu}}(1 + \alpha\beta) - \alpha\overrightarrow{m}(x) - \alpha\beta D\overrightarrow{m}(x) + \mathrm{i}\overrightarrow{j}(x). \tag{44}$$

Thus the complex quaternionic function $\overrightarrow{m}(x)$ satisfies the following equation:

$$\left(D+\frac{\alpha}{(1+\alpha\beta)}\right)\overrightarrow{m}(x)=\frac{\rho(x)}{\epsilon\sqrt{\mu}}+\mathrm{i}\frac{\overrightarrow{j}(x)}{(1+\alpha\beta)}. \tag{45}$$

By analogy of equation (45), we obtain the equation for $\overrightarrow{n}$ as

$$\left(D-\frac{\alpha}{(1-\alpha\beta)}\right)\overrightarrow{n}(x)=\frac{\rho(x)}{\epsilon\sqrt{\mu}}-\mathrm{i}\frac{\overrightarrow{j}(x)}{(1-\alpha\beta)}. \tag{46}$$

On using equations (35) and (41), the continuity equation is described in the following manner:

$$\operatorname{div}\overrightarrow{j_e}=\mathrm{i}\frac{\alpha\rho_e}{\varepsilon\sqrt{\mu}} \qquad \operatorname{div}\overrightarrow{j_m}=\mathrm{i}\frac{\alpha\rho_m}{\upsilon\sqrt{\varepsilon}}. \tag{47}$$

Introducing the notations

$$\alpha_1=\frac{\alpha}{(1+\alpha\beta)} \qquad \alpha_2=\frac{\alpha}{(1-\alpha\beta)} \tag{48}$$

and using equations (47) and (48), we may write the differential equations (45) and (46) in the following forms i.e.,

$$\begin{aligned}(D+\alpha_1)\overrightarrow{m}(x)&=-\mathrm{i}\frac{\operatorname{div}\overrightarrow{j}(x)}{\alpha}-\mathrm{i}\frac{\overrightarrow{j}(x)\alpha_1}{\alpha}\\&=-\frac{\mathrm{i}}{\alpha}(\alpha_1\overrightarrow{j}(x)+\operatorname{div}\overrightarrow{j}(x))\\(D-\alpha_2)\overrightarrow{n}(x)&=\frac{\mathrm{i}}{\alpha}(\alpha_1\overrightarrow{j}(x)-\operatorname{div}\overrightarrow{j}(x)).\end{aligned} \tag{49}$$

In the above equations if we put $\beta=0$, we get the quaternionic form of the generalized Maxwell's equation of dyons in the absence of the chiral medium. In general, the wavenumbers α_1 and α_2 are different and physically characterize the propagation of waves of opposing circular polarizations.

6. Discussion

In the fore-going analysis, equations (37), showing the dependence of the electric displacement vector and the magnetic induction vector on the electric and magnetic fields, do not take into account the chirality of a medium. Instead, they depend upon only on the conductivity, electric permittivity and magnetic permeability of the medium. Chirality is described as the asymmetry in the molecular structure where a molecule is chiral if it cannot be superimposed onto its mirror image. The presence of chirality results in the rotation of electromagnetic fields and its observable, particularly in the microwave range even for the case of a particle consisting electric and magnetic charges (i.e. a dyon). Such experimental observations may be used in physical chemistry to characterize a molecular structure. The present theory of generalized electrodynamics of dyons leads the connection between the mechanical parameters with the chirality and dielectric properties of the brain tissue considered as a bioplasma. Hence the proposal for dyonic bioplasma is being considered. Our theory reduces to the theories described earlier [22–25, 29–32] for the case of electric charge in the absence of magnetic monopole on dyon and consequently the theories of pure monopole described from duality in the absence of electric charge on dyons.

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